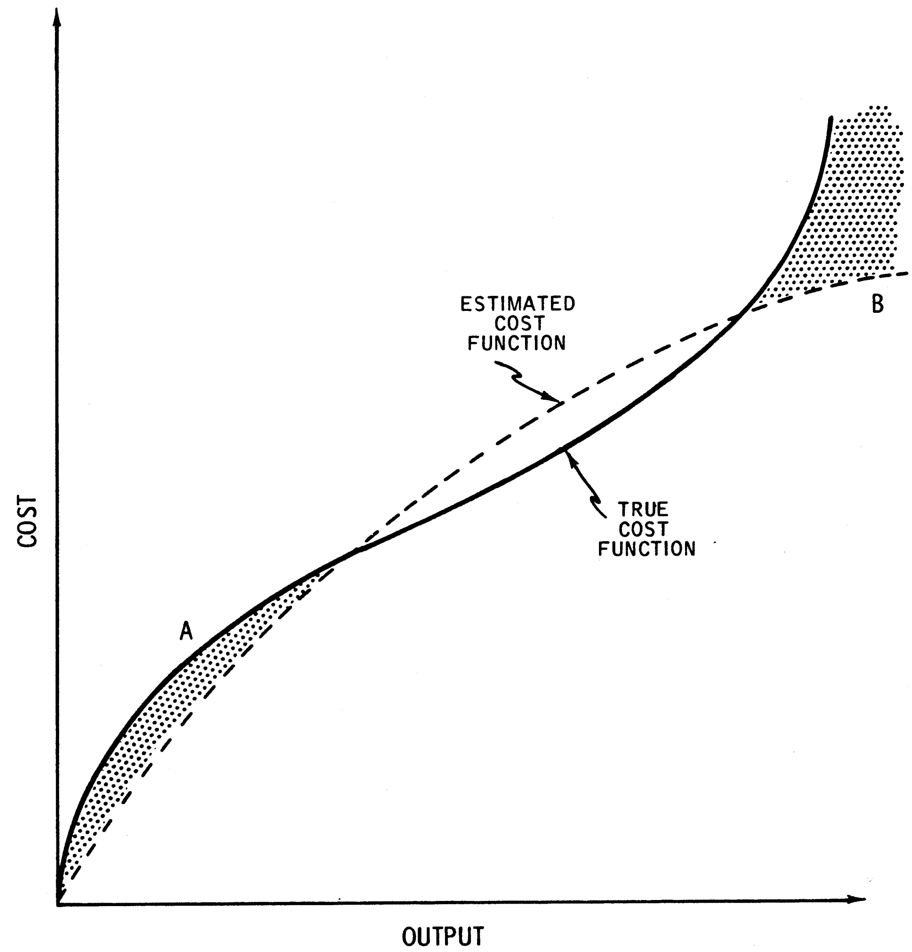

FIG. 3

If the true cost function is not linear in logarithms, we can either fit an over-all function that reflects this fact or attempt to approximate the actual function by a series of segments of functions linear in logarithms. Because of fitting difficulties and the problem of determining the form in which factor prices enter the cost function, I initially chose the latter course. Firms, arrayed in order of ascending output, were divided into 5 groups containing 29 observations each. A list of the firms used in the analysis appears in Appendix C. The results of fitting five separate regressions of the form indicated by Model A are given in lines IIIA through IIIE of Table 3 and the corresponding implications for the parameters in the production function in lines IIIA through IIIE of Table 4. Similar results for regressions of the form indicated by Model B are presented in lines VIA through VIE of Tables 5 and 6.

The results of these regressions with respect to returns to scale are appealing: Except for statistically insignificant reversal between groups C and D, returns to scale diminish steadily, falling from a high of better than 2.5 to a low of slightly less than 1, which indicates increasing returns at a diminishing rate for all except the largest firms in the sample. However, in the case of regressions III, the elasticity of output with respect to capital price behaves very erratically from group to group and has the wrong sign in groups A and E; in regressions VI the elasticity of output behaves erratically, both with respect to labor and with respect to capital, having the wrong sign in groups B and C for the former and in group D for the latter.

Analyses of covariance for regressions III and VI, compared with the over-all regressions I and V, respectively, gave F-ratios of 1.569 and 1.791 in that order. With 141 and 125 degrees of freedom, these ratios are significant at better than the 99 per cent level. Thus, breaking the sample into five groups significantly reduces the residual variance. However, because of the erratic behavior of the coefficients of independent variables other than output, it appears that we may have gone too far. Regressions III and VI are based on the assumption that *all* coefficients differ from group to group. Economically, this may be interpreted as the hypothesis of *non-neutral variations in returns to scale*; i.e., scale affects not only returns to scale but also marginal rates of substitution.

A halfway house between the hypothesis of no variation in returns to scale with output level and the hypothesis of non-neutral variations in scale is the hypothesis of *neutral variations in returns to scale.* A general test of this hypothesis is equivalent to testing the hypothesis that the coefficients for the various prices in the individual group regressions are the same for all groups while allowing the constant terms and the coefficients of output to differ.[8] The hypothesis of neutral variations in returns to scale is tested in this way only in the context of Model A. The regression results are presented in lines IVA through IVE of Table 3 and their implications for the production function in Table 4. An analysis of covariance comparing regressions III and IV gives an F-ratio of 1.576. With 133 and 125 degrees of freedom, a ratio this high is significant at better than the 99 per cent level; hence, we cannot confidently reject the hypothesis of non-neutral variations in returns to scale on statistical grounds alone with this test. Examining the results derived from regressions IV, however, we find that the degree of returns to scale steadily declines with output until, for the group consisting of firms with the largest outputs, we find some evidence of diminishing returns to scale.[9] Furthermore, the elasticities of output with

[8] For a generalized Cobb-Douglas the marginal rate of substitution between x_i and x_j is

$$\frac{\partial y/\partial x_i}{\partial y/\partial x_j} = \frac{a_i/a_j}{x_i/x_j}.$$

Hence, if the ratio of a_i to returns to scale, r, is restricted to be the same for each output group, the marginal rates of substitution will be invariant with respect to output level at each given factor ratio.

[9] Note, however, that the estimated value is insignificantly different from one, so that we cannot reject the hypothesis of constant returns to scale for this group of firms.

respect to the various input levels are all of the correct sign and of reasonable magnitude, although I still feel that the elasticity with respect to capital is implausibly low.[10] Thus, on economic grounds, one might tentatively accept the hypothesis of neutral variations in returns to scale.

If one accepts the hypothesis of neutral variations in returns to scale, a somewhat more refined analysis is possible, since we may then treat the degree of returns to scale as a continuous function of output. That is, instead of grouping the firms as we did previously, we estimate a cost function of the form

$$C = K + \frac{1}{r(Y)} Y + \frac{a_1}{r} P_1 + \frac{a_2}{r} P_2 + \frac{a_3}{r} P_3 , \tag{12}$$

where $r(Y)$, the degree of returns to scale, is a function of the output level. Since neutral variations in returns to scale are assumed, the coefficients of the prices are unaffected. A preliminary graphical analysis indicated that returns to scale as a continuous function of output might be approximated by a function of the form

$$r(y) = \frac{1}{\alpha + \beta \log y} . \tag{13}$$

Thus, instead of regressions of the form suggested by (10) or (11), we fit

$$C - P_3 = K + \alpha Y + \beta Y^2 + \frac{a_1}{r} [P_1 - P_3] + \frac{a_2}{r} [P_2 - P_3] + V \tag{14}$$

(Model C)
and

$$C = K' + \alpha Y + \beta Y^2 + \frac{a_1}{r} P_1 + \frac{a_3}{r} P_3 + V \tag{15}$$

(Model D).

The results obtained for regressions based on Model C and Model D are reported in Table 7 for regressions VII and VIII, respectively. The implications of these results for the production function are given in Table 8. Note that returns to scale and the other parameters have been computed at five output levels only, so that the results in Table 8 may be readily compared with those in Tables 4 and 6.

Perhaps the most striking result of the assumption of continuously and neutrally variable returns to scale of the form suggested in (13) is the substantial increase in our estimate of the degree of returns to scale for firms in the three largest size groups. Whereas before, we found nearly

[10] See p. 179.

TABLE 7

RESULTS FROM REGRESSIONS BASED ON MODELS C AND D FOR 145 FIRMS IN 1955; CONTINUOUS NEUTRAL VARIATIONS IN RETURNS TO SCALE

Model C: Dependent Variable Was $C - P_3$

Regression No.	Coefficient				R^2
VII	Y	Y^2	$P_1 - P_3$	$P_2 - P_3$	
	0.151 (±.062)	0.117 (±.012)	0.498 (±.161)	0.062 (±.151)	0.958

Model D: Dependent Variable Was C

Regression No.	Coefficient				R^2
VIII	Y	Y^2	P_1	P_3	
	0.137 (±.064)	0.118 (±.013)	0.279 (±.224)	0.255 (±.054)	0.952

Figures in parentheses are the standard errors of the coefficients.

constant returns to scale, it now appears that they are increasing.[11] In addition, all the coefficients in both analyses are of the right sign, and the results based on Model D yield results of plausible magnitude for the elasticity of output with respect to capital as compared with the elasticities with respect to labor and fuel. Analyses of covariance, comparing regressions VII and I with regressions VIII and V, yield F-ratios of 1.631 and 9.457, respectively; both are highly significant, with 141 and 140 degrees of freedom. A comparison of regression VII with regression III yields an F-ratio of 1.032, which, though not significant, does suggest that neutral variations in returns to scale of the form used are indistinguishable from non-neutral. Hence the hypothesis of neutral variations in returns to scale may be accepted both on economic grounds and on grounds of simplicity.

[11] Using the variance-covariance matrix for the coefficients in (14) or (15), one could easily compute, for a given y, a conditional standard error for $1/r$, which could then be used to test whether $1/r$ were significantly less than one (i.e., whether the finding of increasing returns was statistically significant). Unfortunately, the regression program used did not print out the inverse of the moment matrix, so this test could not be made. But there is little doubt, in view of the extremely small standard errors of the estimated α and β, that such a test would have shown the increasing returns found to be statistically significant.

TABLE 8

RETURNS TO SCALE AND ELASTICITIES OF OUTPUT WITH RESPECT TO VARIOUS INPUTS DERIVED FROM RESULTS PRESENTED IN TABLE 7 FOR 145 FIRMS IN 1955

Regression VII (Model C)

Group	Returns to Scale[a]	Elasticity of Output with Respect to[a]		
		Labor	Capital	Fuel
A	2.92	1.45	0.18	1.29
B	2.24	1.12	0.14	0.98
C	1.97	0.98	0.12	0.87
D	1.84	0.92	0.11	0.81
E	1.69	0.84	0.10	0.75

Regression VIII (Model D)

Group	Returns to Scale[a]	Elasticity of Output with Respect to[a]		
		Labor	Capital	Fuel
A	3.03	0.85	1.41	0.77
B	2.30	0.64	1.07	0.59
C	2.01	0.56	0.94	0.51
D	1.88	0.52	0.88	0.48
E	1.72	0.48	0.80	0.44

[a] Evaluated at the median output for each group.

3. Conclusions and Prospects

The major substantive conclusions of this paper are that

1. There is evidence of a marked degree of increasing returns to scale at the firm level; but the degree of returns to scale varies inversely with output and is considerably less, especially for large firms, than that previously estimated for individual plants.

2. Variation in returns to scale may well be neutral in character; i.e., although the scale of operation affects the degree of returns to scale, it may